

Online Appendix

Proof of Proposition 1

By backward induction, B must accept if $(1 - x)V_B > (1 - p)V_B - c_B$, or $x < p + \frac{c_B}{V_B}$. By analogous argument, B rejects if $x > p + \frac{c_B}{V_B}$. B's choice when indifferent is immaterial because such a type has measure zero.

We can immediately eliminate two sets of demand strategies for A. First, it will never demand $x > p + \frac{c_B}{V_B}$. Doing so yields war with certainty for a payoff of $pV_A - c_A$. In contrast, it could just demand $x = p$ and guarantee itself a payoff of pV_A . Second, it will never demand $x < p + \frac{c_B}{V_B}$. All types of B accept. But A could deviate to any value between that x and $p + \frac{c_B}{V_B}$. All types of B also accept those demands, but each increases A's share of the bargain.

Thus, the only other values left to consider are $x \in \left[p + \frac{c_B}{V_B}, p + \frac{c_B}{V_B} \right]$. Given B's best response, the probability that B rejects such a demand is the probability that $x > p + \frac{c_B}{V_B}$, or $c_B < V_B(x - p)$. Routing this through the CDF, that probability equals $F(V_B(x - p))$. As such, A's objective function is:

$$F(V_B(x - p))(pV_A - c_A) + (1 - F(V_B(x - p)))xV_A$$

The first order condition of this is:

$$\frac{\partial}{\partial x} \left(F(V_B(x - p)) \left(p - \frac{c_A}{V_A} \right) + (1 - F(V_B(x - p)))x \right) = 0$$

$$\frac{1}{V_B \left(x - p + \frac{c_A}{V_A} \right)} = \frac{f(V_B(x - p))}{1 - F(V_B(x - p))} \quad (1)$$

If such a solution exists, then it is unique. This is because the left hand side decreases in x and the right hand side weakly increases in x .¹ Therefore, A makes that demand.²

¹The right hand side is the hazard rate, which is weakly increasing.

²It is a maximum because, if such a solution exists, the derivative is positive at $x = p + \frac{c_B}{V_B}$

However, it is possible that A's utility decreases at the point $x = p + \frac{c_B}{V_B}$, the maximum value of x that all types accept. Substituting, this arises when:

$$\frac{1}{\frac{V_B c_A}{V_A} + c_B} < f(c_B)$$

In this case, A's optimal demand is $x = p + \frac{c_B}{V_B}$.

Proof of Proposition 2

For proof by contradiction, suppose that the probability of war is strictly greater under V_B'' than under V_B' , where $V_B'' > V_B'$. Let c_B' be the type that A's optimal demand makes indifferent under V_B' and c_B'' be the type that A's optimal demand makes indifferent under V_B'' . The probability of war for each of these cases is $F(c_B')$ and $F(c_B'')$. By assumption of the proof by contradiction, we have $F(c_B'') > F(c_B')$. Because $F(c_B)$ is strictly increasing, this implies $c_B'' > c_B'$.

The proof for Proposition 1 showed that there is a unique optimal demand. Because making the c_B' type indifferent is uniquely optimal under V_B' , A's utility for the corresponding demand $p + \frac{c_B'}{V_B'}$ must be strictly greater than its utility for demanding $p + \frac{c_B''}{V_B'}$. That is:

$$\begin{aligned} F(c_B') \left(p - \frac{c_A}{V_A} \right) + (1 - F(c_B')) \left(p + \frac{c_B'}{V_B'} \right) &> F(c_B'') \left(p - \frac{c_A}{V_A} \right) + (1 - F(c_B'')) \left(p + \frac{c_B''}{V_B'} \right) \\ V_B' &> \frac{c_B''(1 - F(c_B'')) - c_B'(1 - F(c_B'))}{\frac{c_A}{V_A}(F(c_B'') - F(c_B'))} \end{aligned} \quad (2)$$

Likewise, because c_B'' is uniquely optimal under V_B'' , A's utility for the corresponding demand $p + \frac{c_B''}{V_B''}$ must be strictly greater than its utility for demanding $p + \frac{c_B'}{V_B''}$. That is:

$$F(c_B'') \left(p - \frac{c_A}{V_A} \right) + (1 - F(c_B'')) \left(p + \frac{c_B''}{V_B''} \right) > F(c_B') \left(p - \frac{c_A}{V_A} \right) + (1 - F(c_B')) \left(p + \frac{c_B'}{V_B''} \right)$$

$$V_B'' < \frac{c_B''(1 - F(c_B'')) - c_B'(1 - F(c_B'))}{\frac{c_A}{V_A}(F(c_B'') - F(c_B'))} \quad (3)$$

Stringing Lines 2 and 3 together, we have:

$$V_B'' < \frac{c_B''(1 - F(c_B'')) - c_B'(1 - F(c_B'))}{\frac{c_A}{V_A}(F(c_B'') - F(c_B'))} < V_B'$$

This implies $V_B' > V_B''$. But $V_B'' > V_B'$. Hence a contradiction. Thus, the probability of war must weakly decrease.

Extremely Low Valuations

Here, we sketch what happens when V_B is low enough that some types have a negative value for war, using the uniform distribution from Figure 2. A's optimization problem is the same as before, with an important exception. The solution to the first order condition is $p + \frac{\bar{c}_B}{2V_B} - \frac{c_A}{2V_A}$. However, it is clear that if V_B is sufficiently small, that this exceeds 1. In that case, A's optimal demand is the entire good. As before, the probability of war equals the probability of a drawing a type with cost value lower than the type A's demand makes indifferent. Put differently, for a given demand x , the probability of war is $1 - p - \frac{c_B}{V_B} > 1 - x$, or $c_B < V_B(x - p)$. Substituting $x = 1$ and using the uniform distribution's CDF generates a probability of war equal to $\frac{V_B(1-p) - c_B}{\bar{c}_B - c_B}$. This increases in V_B .

However, sufficiently large changes to V_B shifts A's demand from $x = 1$ to the interior solution. The equilibrium probability of war becomes $\frac{\bar{c}_B - \frac{V_B c_A}{V_A} - 2c_B}{2(\bar{c}_B - c_B)}$, which is decreasing in V_B as Proposition 2 guarantees. This generates the nonmonotonicity in Figure 2.

Extension: Joint Changes to Valuations

We begin with a restatement of Proposition 2's results:

Proposition 3. *Let $\pi'' > \pi'$. Then comparing a π' case to a π'' case, the probability of war weakly decreases if $\frac{V_B(\pi'')}{V_A(\pi'')} \geq \frac{V_B(\pi')}{V_A(\pi')}$.*

The proof is a generalization of the proof for Proposition 2. For proof by contradiction, suppose that the probability of war is strictly greater under π'' than under π' . Let c'_B be the type that A's optimal demand makes indifferent under π' and c''_B be the type that A's optimal demand makes indifferent under π'' . The probability of war for each of these cases is $F(c'_B)$ and $F(c''_B)$. By assumption of the proof by contradiction, we have $F(c''_B) > F(c'_B)$. Because $F(c_B)$ is strictly increasing, this implies $c''_B > c'_B$.

There is still a unique optimal demand. Because making the c'_B type indifferent is uniquely optimal under π' , A's utility for the corresponding demand $p + \frac{c'_B}{V_B(\pi')}$ must be strictly greater than its utility for demanding $p + \frac{c''_B}{V_B(\pi')}$. That is:

$$F(c'_B) \left(p - \frac{c_A}{V_A(\pi')} \right) + (1 - F(c'_B)) \left(p + \frac{c'_B}{V_B(\pi')} \right) > F(c''_B) \left(p - \frac{c_A}{V_A(\pi')} \right) + (1 - F(c''_B)) \left(p + \frac{c''_B}{V_B(\pi')} \right)$$

$$V_B(\pi') \left(\frac{c_A}{V_A(\pi')} \right) > \frac{c''_B(1 - F(c''_B)) - c'_B(1 - F(c'_B))}{F(c''_B) - F(c'_B)} \quad (4)$$

Likewise, because c''_B is uniquely optimal under $V_B(\pi'')$, A's utility for the corresponding demand $p + \frac{c''_B}{V_B(\pi'')}$ must be strictly greater than its utility for demanding $p + \frac{c'_B}{V_B(\pi'')}$. That is:

$$F(c''_B) \left(p - \frac{c_A}{V_A(\pi'')} \right) + (1 - F(c''_B)) \left(p + \frac{c''_B}{V_B(\pi'')} \right) > F(c'_B) \left(p - \frac{c_A}{V_A(\pi'')} \right) + (1 - F(c'_B)) \left(p + \frac{c'_B}{V_B(\pi'')} \right)$$

$$V_B(\pi'') \left(\frac{c_A}{V_A(\pi'')} \right) < \frac{c''_B(1 - F(c''_B)) - c'_B(1 - F(c'_B))}{F(c''_B) - F(c'_B)} \quad (5)$$

Stringing Lines 4 and 5 together, we have:

$$V_B(\pi'') \left(\frac{c_A}{V_A(\pi'')} \right) < \frac{c''_B(1 - F(c''_B)) - c'_B(1 - F(c'_B))}{F(c''_B) - F(c'_B)} < V_B(\pi') \left(\frac{c_A}{V_A(\pi')} \right)$$

This implies:

$$\frac{V_B(\pi'')}{V_A(\pi'')} < \frac{V_B(\pi')}{V_A(\pi')}$$

But this is contradicted for the parameters covered in Proposition 3. This has the same interpretation that the main text offered. When $\frac{V_B(\pi'')}{V_A(\pi'')} \geq \frac{V_B(\pi')}{V_A(\pi')}$, the change to V_B has a larger effect than the change to V_A . This means that the reduction to the peace premium has a greater impact on A's bargaining decision than A's less-internalized cost of war.

Extension: Uncertainty over Power

In this variant, Nature begins by drawing B as the \bar{c}_B type with probability q and as the \underline{c}_B type with probability $1 - q$. Play remains identical, except that A wins against the \bar{c}_B type with probability \bar{p} and wins against the \underline{c}_B with probability \underline{p} .

Only two demands can be optimal for A: $\bar{p} + \frac{\bar{c}_B}{V_B}$ and $\underline{p} + \frac{\underline{c}_B}{V_B}$. All other demand give an unnecessary concession or assuredly result in war. The former demand results in acceptance with probability q and in rejection by the low cost type with probability $1 - q$. The latter demand results in acceptance with certainty. As such, A prefers the risky demand if:

$$q \left(\bar{p} + \frac{\bar{c}_B}{V_B} \right) + (1 - q) \left(\underline{p} - \frac{c_A}{V_A} \right) > \underline{p} + \frac{\underline{c}_B}{V_B}$$

$$q > \frac{\frac{c_A}{V_A} + \frac{\underline{c}_B}{V_B}}{\bar{p} - \underline{p} + \frac{c_A}{V_A} + \frac{\bar{c}_B}{V_B}}$$

Thus, A makes the risky demand under fewer circumstances when the right hand side increases in V_B . Taking the respective derivative yields:

$$\bar{p} - \underline{p} < \left(\frac{c_A}{V_A} \right) \left(\frac{\bar{c}_B}{\underline{c}_B} - 1 \right)$$

The right hand side is strictly positive. As such, for sufficiently low differences in power between types ($\bar{p} - \underline{p}$), A makes the safer demand under a wider variety of circumstances. In turn, the probability of war weakly decreases, consistent with the main article's logic.